School	*Candidate's Name (PLEASE PRINT)*

WINCHESTER
COLLEGE

Entrance Examination

Mathematics

2021

Time allowed: 1 hour 30 minutes

Total marks: 100

CALCULATORS ARE NOT ALLOWED.

You may use a pencil for diagrams. You should show all your working so that credit may be given for partially correct answers.

Do not be discouraged if you do not finish. If you get more than 60 marks, you will have done well.

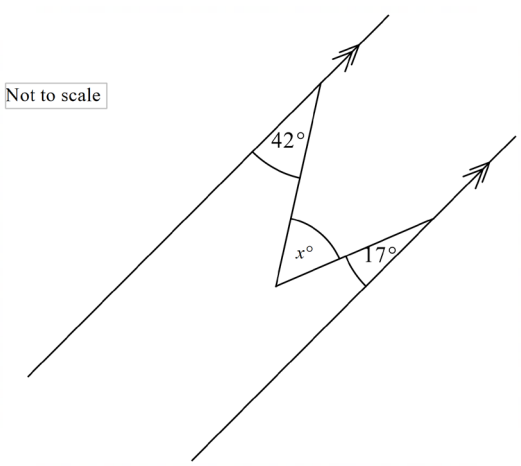
1.	Complete: a) $15^2 =$	b) $2\times2\times2\times2\times5\times5\times5 =$	[1] [1]
	c) $422-500+579 =$	d) $\dfrac{2+48}{2+3} =$	[1] [1]
	e) $\sqrt{196}\times\sqrt{196} =$	f) $\sqrt{13^2-5^2} =$	[1] [1]
	g) $98\div0.014 =$	h) $99+49\times99 =$	[1] [1]

2.	a) Find $\frac{1}{8}$ of £1024.	b) Find 76% of £25.	[1] [1]
	c) Find $0.\dot{7}$ of £117.	d) Evaluate $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of £556.	[1] [1]
	e) The price of a jumper was £49. It was reduced by 20% then increased by 25% and finally reduced by 30%. What is the new price of the jumper?	f) Evaluate $185 \times 0.32 + 1.5 \times 3.2$.	[2] [2]

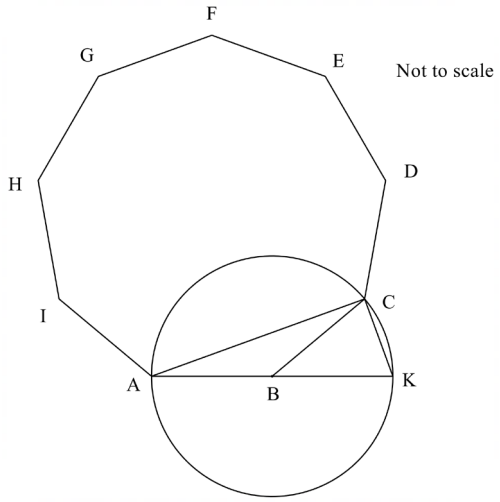
3.	Find in its simplest form: a) $87 \times \dfrac{87+87}{87}$	b) $\dfrac{2}{77}+\dfrac{1}{143}$	[1] [2]
	c) $51 \div 4\dfrac{1}{4}$	d) $\left(\dfrac{(-18)^3}{(-9)^3}+\dfrac{(-14)^3}{7^3}+1\right) \div \dfrac{(-6)^2}{(-12)^2}$	[2] [3]

4.

a) The lines below are parallel. Find x. [2]



b) The nine-sided polygon ABCDEFGHI is regular.
The circle has centre B and diameter AK.



Find the following angles:

i) BAC [2]

ii) AKC [1]

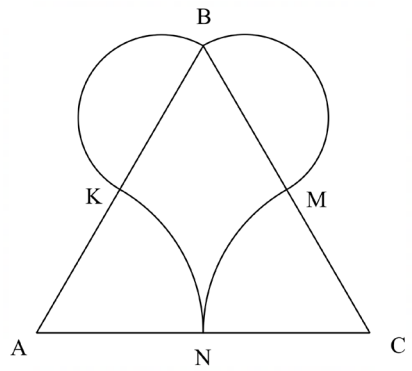
iii) ACK [1]

5.	$a=3$, $b=-5$ and $c=-11$. a) Evaluate b^2-4ac.	b) Evaluate $(a+b)(a^2-ab+b^2)$.	[2] [2]
	c) Find $\left(a+\frac{1}{a}\right)^2$ as a mixed number in its simplest form. Show it is not equal to $a^2+\frac{1}{a^2}$.	d) Evaluate $1+a(2+3(b-(121\div c)))$.	[3] [2]

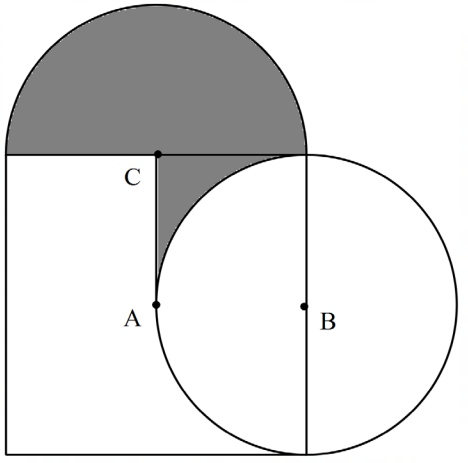
6.

a) The diagram shows an equilateral triangle with side length 6 cm. K, M and N are midpoints of the triangle's sides.

The heart shape is made of two semicircles (with diameter BK and BM) and two arcs KN and MN. KN is the arc of the circle centred at A and MN is the arc of the circle centred at C. The perimeter of the heart shape is $k\pi$. Find k. [3]

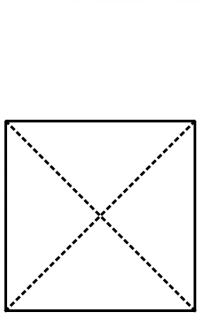


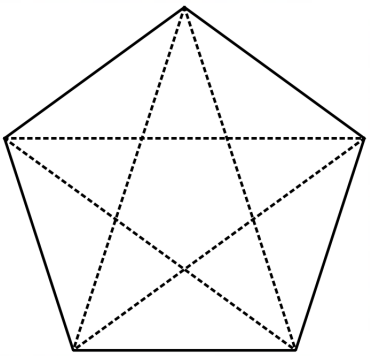
b) The shape below is made up of a square with centre A, a circle with centre B and a semicircle with centre C. What fraction of the shape is shaded? [3]



7.	a) Start with x. Add four. Multiply by two. Subtract one. Write down an expression that corresponds to these instructions and simplify your answer.	b) Expand $(5a)^2 - 3(2a^2 - 15)$ and simplify.	[2] [2]
	c) $\frac{16016}{b^3 - 11} = 1001$. Find b.	d) Solve $\frac{10}{1 - \frac{1}{1+x}} = -5$.	[2] [2]

8.





a) A square has 2 diagonals, and a pentagon has 5 (see above). How many diagonals does a regular hexagon have? [2]

b) How many diagonals would a 10-sided regular polygon have? [2]

c) Holly creates a regular polygon that has exactly 170 diagonals. How many sides does this polygon have? [2]

9. A mother has five children, Amy, Boris, Cara, David and Ezra. She says to each of them "I will give you an amount of money of your choosing up to £1000. In addition, every year I shall give you one tenth of the amount of the £1000 that you did *not* take."

Amy chooses to receive £300. Next year she receives one tenth of £700 which is £70. The year after she receives another £70, and so after two years she has received, in total, £440.

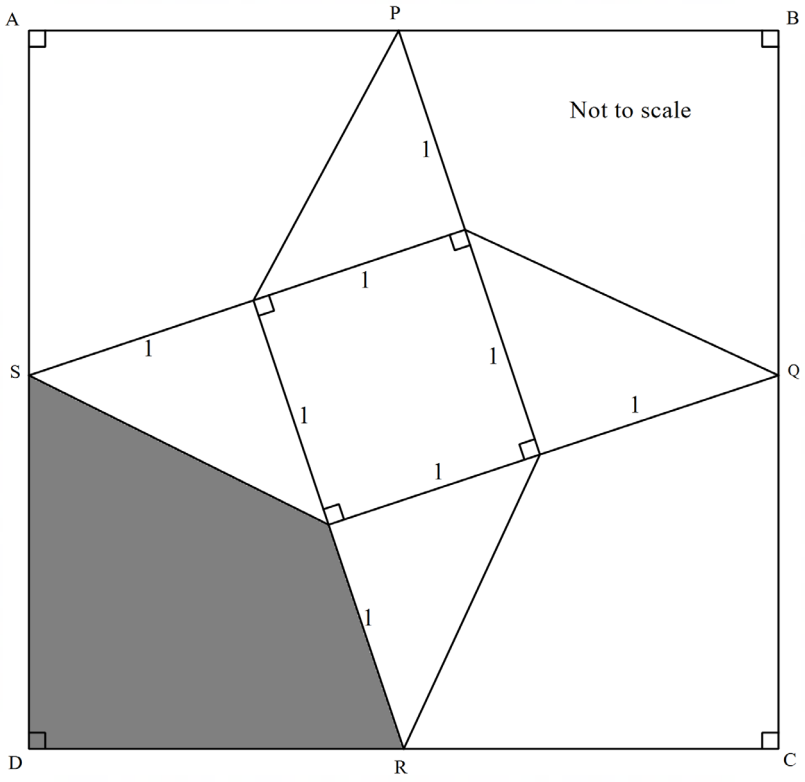
a) Boris chooses to initially receive £200. How much does he receive, in total, after three years? [2]

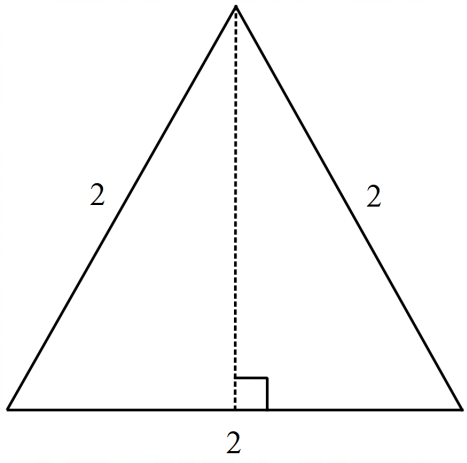
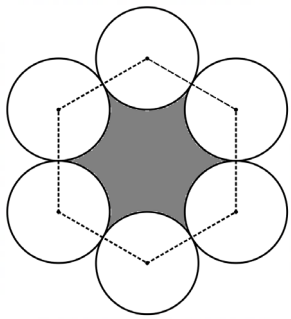
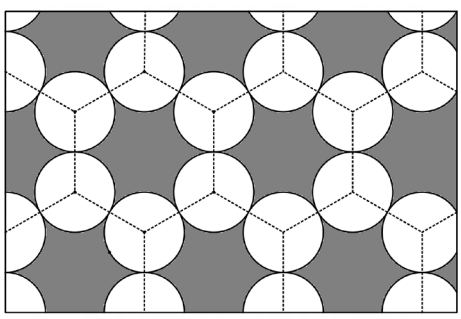
b) Cara chooses to initially receive £400. After how many years has she received, in total, £760? [2]

	c) After nine years David wants to have the maximum possible amount. How much should he choose to initially receive?	[2]
	d) After eleven years Ezra wants to have the maximum possible amount. How much should he choose to initially receive?	[2]

10.	There are two ways to express 6 as a sum of two different positive whole numbers written in increasing order, namely $1+5$ and $2+4$.	
	a) List all the ways to express 11 as a sum of two different positive whole numbers written in increasing order.	[2]
	b) How many ways can 2021 be expressed as a sum of two different positive whole numbers written in increasing order?	[2]
	c) List all the ways to express 9 as a sum of three different positive whole numbers written in increasing order.	[2]

	d) Alysha wants to express 99 as a sum of three different positive whole numbers written in increasing order, with the additional constraint that the mean of the numbers must be the same as their median. How many ways can this be done?	[2]
	e) Bob wants to express 22 as a sum of four different numbers written in increasing order, with the additional constraint that the mean of the numbers must be the same as their median. How many ways can this be done?	[3]

11.	In the diagram below ABCD is a square and P, Q, R and S are the midpoints of the square's edges. A line of length 2 extends from each midpoint to form a square with side length 1 inside the larger square. Find the shaded area.  	[5]

12.	a) Find the area of the equilateral triangle with side length 2.  	[2]
	b) Six circles of radius 1 are arranged in a regular hexagon as shown below. The area of the shaded region is $a\sqrt{3}+b\pi$. Find a and b. 	[3]
	c) The diagram shows a pattern of circles and hexagons. This pattern is extended so that it covers a large rectangular area. Ignoring the effects of the edges of the rectangle, what is the ratio of circles to hexagons used? 	[2]

13. A square grid of positive integers is called *good* if the following two conditions hold:

- Each number in the grid is different and is a product of distinct odd primes less than 14.
- If two numbers are horizontally or vertically adjacent in the grid, then one can be transformed into the other by dividing by a prime and then multiplying by a different prime.

Below are two examples of good 2×2 grids.

33	55
15	35

65	39
55	33

a) Copy and complete the good grid below. [4]

	77	
55		33
	91	

b) Find a number that is the product of two distinct odd primes less than 14 but does not appear in the grid. [1]

c) Copy and complete the good grid below. [4]

	195	
273		455
	165	

d) Find a number that is the product of three distinct odd primes less than 14 but does not appear in the grid. [1]

(END OF PAPER)